Int. J. of Geol. & Earth Sci., 2018

International Journal of Geology and Earth Sciences

ISSN 2395-647X www.ijges.com
Vol. 4, No. 2, June 2018

Research Paper

USING FORWARD MODELING IN CALCULATING THE SHORTEST DISTANCE BETWEEN THE SHOT POINT AND THE FIRST GEOPHONE THAT SENSES THE REFRACTED HEAD WAVES

Alhussein Adham Basheer[1]* and Adel D M Kotb[2]

*Corresponding Author: Alhussein Adham Basheer ✉ alhussein.adham@science.helwan.edu.eg

Received on: 13th April, 2018 Accepted on: 17th May, 2018

Due to the importance of seismic waves and their multiple uses, especially in near-surface studies, the dependence of the data reverse modeling on direct modeling, and the reliance of the data accuracy on the array of geophones, simple mathematical forward modeling and the relationship among time, velocity and distance were used to formulate an equations and graphical relationships. They specify the shortest distance between shot-point and its first nearest geophone that can sense the critical refracted head waves in the cases of two and three layers. These relationships show that the largest distance between the source of the waves (shot-point) and the nearest geophone, which can detect the refracted head waves, should not be more than (nine times of the first layer's thickness) in the case of two layers, and it is doubled twice for each layer that is more than two subsurface layers above the refraction surface. This distance is steadily reduced with the variance of velocities increases. The total penetrated depth not only depends on the distance amongst shot point, the first and the last geophones, but also it depends on the differences of velocities between the layers.

Keywords: Forward modeling, Shallow seismic refraction, Critical refracted head waves

INTRODUCTION

Long times ago, seismic methods are considered as an effectual one for inspecting the near subsurface for a variety of applications, including groundwater studies, mineral exploration (Hobson *et al.*, 1970), geotechnical calculations, engineering problems, archeology, environmental investigations, hydrogeological researches, seismic hazard valuations, etc. (Shti Velman 2003).

Shallow seismic refraction tool offers a reliable method of surveys in which the penetrated depths to essential refractors and the velocities analysis in the refractors are comprehensively demanded (Alan and Steve, 2011; and Steve and Alan, 2011). The calculation of the depth is built on Early's concept (1931), which is called (time-depth) that differs from (delay-time) to some extent. The computation of the single (time-depths) and of

[1] Lecturer of Geophysics, Geology Department, Faculty of Science, Helwan University, 11790 Ain Helwan, Cairo, Egypt.
[2] Geology Department, Faculty of Science, Helwan University, 11790 Ain Helwan, Cairo, Egypt.

the modified travel times within the refractor simply necessitates a few numerical calculation (Hawkins, 1961). Therefore, the tool is so appropriate for regular survey techniques in which combined velocity terms are measured for overburden multilayers (Vrettos, 1996).

Forward modeling is beneficial in field of geophysics to investigate the subsurface of the earth. Not only is it advantageous as a method for data interpretation in a research milieu, but also it is used to increase the physical understanding in an educational scene (Butlera *et al.*, 2014). Their rule is very important in the accurateness and the clarity of both measured and interpreted data. In this way, the accuracy of the inversion will be highly achieved (Burger, 1996).

The geological models of the earth are utilized to simulate seismic field experiments in the forward numerical modeling of shallow seismic refraction data (Kearey and Brooks, 1984). The dimension of these geological models can be one; two, or three, and they involve depth horizons and accompanying P and S wave velocities (Fagin, 1991).

The method chosen to solve the problem is not the major controlling factor on the accurateness of a forward model, but it is the unsuitable usage of necessary one/two dimensional models of the geology that highly affect such accurateness (Susana *et al.*, 2018). The limiting factor for the dimensional model is not the dimension of the data, but it is the complexity of the geology (Derecke, 1981). The inappropriate use of the correct dimension models can increase the amplitude error, the noticeable travel-time, and missing or misinterpreted arrivals (Andrey *et al.*, 2013).

When seismic velocity rises at an interface (V2>V1), and the angle of incidence is risen to reach to more than zero, the transmitted P wave will ultimately arise to 90°. Refracted waves move along the upper margin of the lower medium (Hilterman, 1970; Kelley *et al.*, 1976; Hubral, 1977; and Fagin, 1991). The interaction of these waves with the interface creates secondary sources that yield an up going wave-front that is called a head wave, by Huygen's principle (Gabriela, 2017). The ray accompanying with this head wave comes from the interface at the critical angle. This occurrence is the base of the refraction surveying tool (Fowler, 1990).

Determining the actual site where the head wave reaches the surface, is considered one of the most mysterious transactions in the interpretation of shallow seismic refraction data (Dijksterhuis, 2004). This distance between the shot-point, as a wave's source, and the first geophone, which can detect the head waves, has not theoretically been taken into consideration. Moreover, it is very important in determining the distances between each geophone and the waves' source. It also can help in determining the depth to which the measurement can penetrate.

In this paper, the important factor of the shortest distance between the shot-point and the first geophone that can record the head wave, in relationship with layers' thickness and different velocities, is modeled by forward method to make correct field array of geophones that give precise measured data. This will raise the accuracy of interpretation and will give a new mathematical principle for the depth extent to which refractive waves can reach depending on the field acquisition.

MATHEMATICAL RELATIONS

A signal, which is not unlike a sound pulse, is spread into the surface of the earth. According to Snell's law and Huygen's principle, the ray accompanying with head wave comes from the interface at the critical angle (Figure 1).

Two Layers Case

In the case of the ground that consists of two layers, the head wave can be recorded when the time of direct waves is equal to the time of head wave ($Tx_d = Tx_{r1}$) as in the Figure 2.

$$T_{xd} = \frac{X_{d1}}{v_1} \quad \text{...(E1)}$$

$$T_{xr1} = T_{sa} + T_{ab} + T_{bg} = \frac{X_{sa}}{v_1} + \frac{X_{ab}}{v_2} + \frac{X_{bg}}{v_1} \quad \text{...(1)}$$

by using

$$\frac{X_{sa}}{v_1} = \frac{X_{bg}}{v_1} = \frac{h_1}{v_1 \cos\theta_{c1}} \quad \text{...(2)}$$

and

$$\frac{X_{d1} - 2h\tan\theta_{c1}}{v_2} = \frac{X_{ab}}{v_2} \quad \text{...(3)}$$

then

$$T_{xr1} = \frac{2h_1}{v_1 \cos\theta_{c1}} + \frac{X_{d1} - 2h\tan\theta_{c1}}{v_2} = \frac{2h_1 \cos\theta_{c1}}{v_1} + \frac{X_{d1}}{v_2} \quad \text{...(4)}$$

$$T_{xr1} = \frac{2h_1\sqrt{1 - \sin^2\theta_{c1}}}{v_1} + \frac{X_{d1}}{v_2} \quad \text{...(5)}$$

$$T_{xr1} = \frac{2h_1\sqrt{v_2^2 - v_1^2}}{v_2 v_1} + \frac{X_{d1}}{v_2} \quad \text{...(E2)}$$

from Equations (E1) and (E2)

$$\frac{X_{d1}}{v_1} = \frac{2h_1\sqrt{v_2^2 - v_1^2}}{v_2 v_1} + \frac{X_{d1}}{v_2} \quad \text{...(6)}$$

$$\frac{X_{d1}}{v_1} - \frac{X_{d1}}{v_2} = \frac{2h_1\sqrt{v_2^2 - v_1^2}}{v_2 v_1} \quad \text{...(7)}$$

$$\frac{X_{d1}(v_2 - v_1)}{v_2 v_1} = \frac{2h_1\sqrt{v_2^2 - v_1^2}}{v_2 v_1} \quad \text{...(8)}$$

$$X_{d1} = 2h_1\frac{\sqrt{v_2^2 - v_1^2}}{(v_2 - v_1)} = 2h_1\frac{\sqrt{(v_2 + v_1)(v_2 - v_1)}}{(v_2 - v_1)}$$

$$= 2h_1\sqrt{\frac{v_2 + v_1}{v_2 - v_1}} \quad \text{...(9)}$$

$$X_{d1} = 2h_1\sqrt{\frac{v_2 + v_1}{v_2 - v_1}} \quad \text{...(E3)}$$

Figure 1: (a) Sell's Law, (b) Wave Propagation According to Huygens Principle

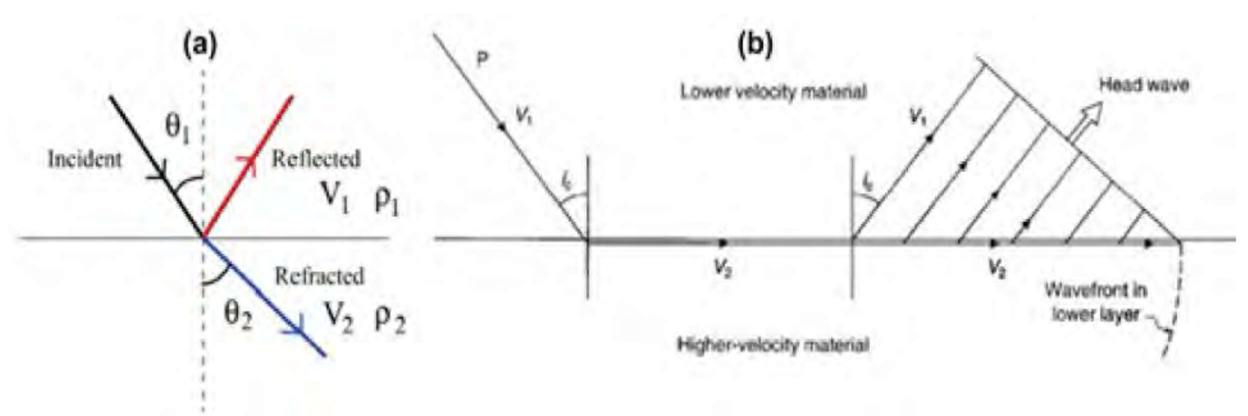
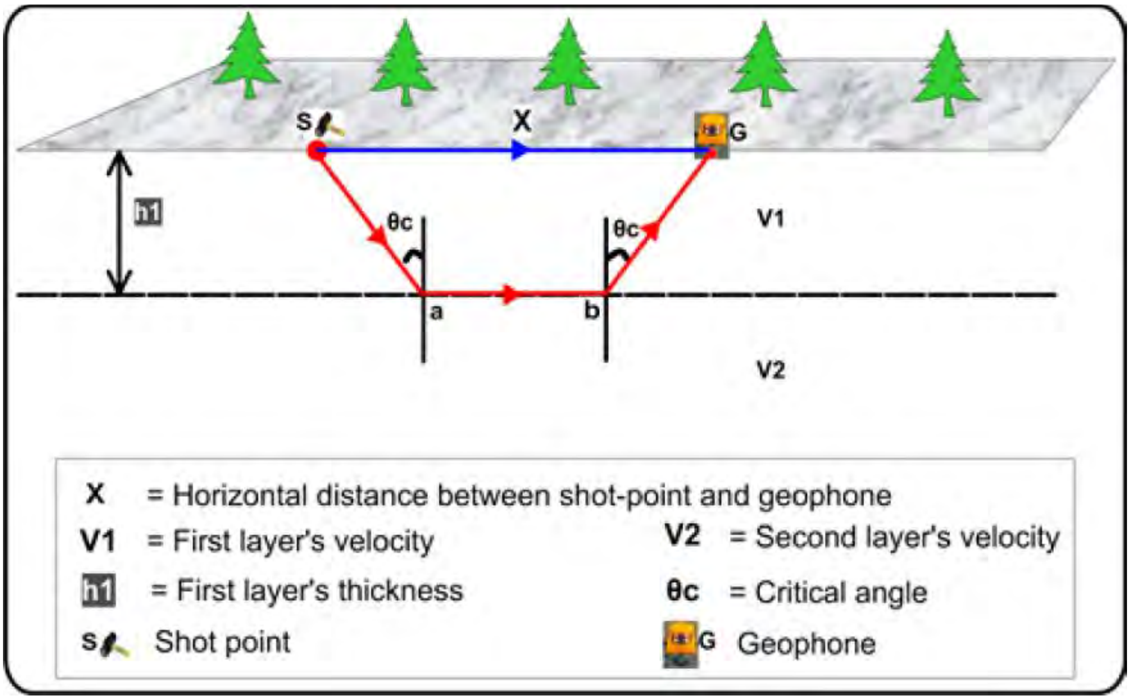


Figure 2: Behavior of Refracted Waves in the Case Two Layers



Three Layers Case

In the case of the ground that consists of three layers, the refracted head wave, which comes from the 2nd interface, can be recorded when its travel time equals to the time of refracted head wave that comes from the 1st interface as in the Figure 3.

$$Tx_{r1} = Tx_{r2} \quad \text{...(E4)}$$

$$T_{xr2} = T_{sa} + T_{ac} + T_{cd} + T_{bc} + T_{bg}$$

In the same vein, from above-mentioned Equation (E2), we can conclude

$$T_{xr2} = \frac{2h_1 Cos\theta_1}{v_1} + \frac{2h_2 Cos\theta_{c2}}{v_2} + \frac{X_{d2}}{v_3} \quad \text{...(10)}$$

$$T_{xr2} = \frac{2h_1 Cos\theta_1}{v_1} + \frac{2h_2\sqrt{v_3^2 - v_2^2}}{v_3 v_2} + \frac{X_{d2}}{v_3} \quad \text{...(11)}$$

by using

$$Cos\theta_1 = \sqrt{1 - Sin^2\theta_1} \quad \text{...(12)}$$

and

$$Sin^2\theta_1 = \frac{v_1^2}{v_2^2} Sin^2\theta_{c2} = \frac{v_1^2}{v_2^2} * \frac{v_2^2}{v_3^2} = \frac{v_1^2}{v_3^2} \quad \text{...(13)}$$

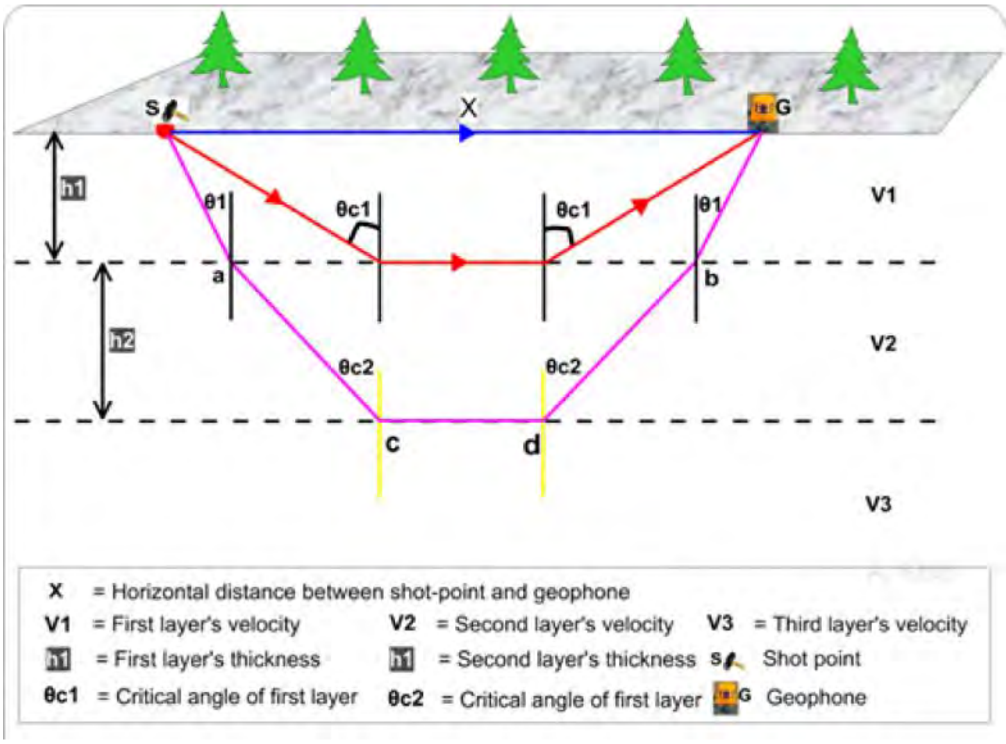
It will be

$$\frac{2h_1\sqrt{v_3^2 - v_1^2}}{v_3 v_1} + \frac{2h_2\sqrt{v_3^2 - v_2^2}}{v_3 v_2} + \frac{X_{d2}}{v_3} \quad \text{...(E5)}$$

Then, compensation in Equations (E2) and (E5), the Equation (E4) can be rewritten as:

$$\frac{2h_1\sqrt{v_2^2 - v_1^2}}{v_2 v_1} + \frac{X_{d2}}{v_2} = \frac{2h_1\sqrt{v_3^2 - v_1^2}}{v_3 v_1} + \frac{2h_2\sqrt{v_3^2 - v_2^2}}{v_3 v_2} + \frac{X_{d2}}{v_3} \quad \text{...(14)}$$

Figure 3: Behavior of Refracted Waves in the Case of Three Layers



$$\frac{X_{d2}}{v_2}-\frac{X_{d2}}{v_3}=\frac{2h_1\sqrt{v_3^2-v_1^2}}{v_3v_1}-\frac{2h_1\sqrt{v_2^2-v_1^2}}{v_2v_1}+\frac{2h_2\sqrt{v_3^2-v_2^2}}{v_3v_2} \quad ...(15)$$

$$\frac{X_{d2}\ (v_3-v_2)}{v_2v_3}=\frac{2h_1v_3v_2\sqrt{v_3^2/v_3^2-v_1^2/v_3^2-v_2^2/v_2^2+v_1^2/v_2^2}}{v_3v_2v_1}+\frac{2h_2\sqrt{v_3^2-v_2^2}}{v_3v_2} \quad ...(16)$$

$$\frac{X_{d2}\ (v_3-v_2)}{v_2v_3}=2h_1\frac{\sqrt{v_1^2/v_2^2-v_1^2/v_3^2}}{v_1}+2h_2\frac{\sqrt{v_3^2-v_2^2}}{v_3v_2} \quad ...(17)$$

$$\frac{X_{d2}\ (v_3-v_2)}{v_2v_3}=2h_1\frac{\sqrt{v_1^2v_3^2-v_1^2v_2^2}}{v_1v_2v_3}+2h_2\frac{\sqrt{v_3^2-v_2^2}}{v_3v_2} \quad ...(18)$$

$$\frac{X_{d2}\ (v_3-v_2)}{v_2v_3}=(2h_1+2h_2)\frac{\sqrt{v_1^2v_3^2-v_1^2v_2^2+v_1^2v_3^2-v_1^2v_2^2}}{v_1v_2v_3} \quad ...(19)$$

$$\frac{X_{d2}\ (v_3-v_2)}{v_2v_3}=\{2(h1+h2)\}\frac{\sqrt{v_1^2v_3^2-v_1^2v_2^2}}{v_1v_2v_3} \quad ...(20)$$

$$X_{d2} = \{2(h1+h2)\}\frac{\sqrt{v_1^2 v_3^2 - v_1^2 v_2^2}}{v_1 v_3 - v_1 v_2} \quad ...(21)$$

$$X_{d2} = \{2(h1+h2)\}\sqrt{\frac{v_1 v_3 + v_1 v_2}{v_1 v_3 - v_1 v_2}} \quad ...(22)$$

by setting $\mathbf{h_1+h_2 = z}$, the equation (22) will be

$$X_{d2} = 2Z\sqrt{\frac{v_3+v_2}{v_3-v_2}} \quad ...(E6)$$

In case of $V_3 = 2V_2$, the Equation (E7) can be written as:

$$X_d = 2z\sqrt{3} \quad ...(E7)$$

In the same context, The X_{dn} for (n) numbers of interfaces can be calculated as follows:

$$X_{dn} = 2(h1+h2+\cdots+hn)\sqrt{\frac{v_n+v_{n-1}}{v_n-v_{2n-1}}} \quad ...(E8)$$

$$X_{dn} = 2\sum_{i=1}^{n} h_i \sqrt{\frac{v_n+v_{n-1}}{v_n-v_{2n-1}}} \quad ...(E9)$$

RESULTS AND DISCUSSION

In the case of two layers, Equation (E3) can be rewritten according to the change in the first layer's velocity, in ratio to the second layer's velocity, as in the Table 1.

Table 1: The Changes in Equation (E3) by Change the First Layer's Velocity in Ratio to the Second Layer's Velocity

Velocity Relation	Rewrite of Equation (E3)
$V_2=3/2v_1$	$X_{d1} = 2h_1\sqrt{5}$
$V_2=2v_1$	$X_{d1} = 2h_1\sqrt{3}$
$V_2=5/2v_1$	$X_d = 2h_1\sqrt{7/3}$
$V_2=3v_1$	$X_d = 2h_1\sqrt{2}$

In the case of three layers, Equation (E3) can be rewritten according to the change in the second layer's velocity, in ratio to the third layer's velocity, as in the Table 2.

Table 2: The Changes in Equation (E3) by Change the Second Layer's Velocity in Ratio to the Third Layer's Velocity

Velocity Relation	Rewrite of Equation (E3)
$V_3=3/2v_2$	$X_{d2} = 2Z\sqrt{5}$
$V_3=2v_2$	$X_{d2} = 2Z\sqrt{3}$
$V_3=5/2v_2$	$X_d = 2\sqrt{7/3}$
$V_3=3v_2$	$X_d = 2\sqrt{2}$

In comparison between Table 2 and the previous Table 1, it can be inferred that in the case of equal velocities above and below the surface of reflection, regardless of the number of layers, the control factor of the distance between shot-point and first geophone, which can actually record the head waves, is the depth from the surface, where the waves are originated, to the surface of the reflection.

Since the crustal rocks have velocities ranging from 350 to 8500 meters per second, the relationship between curves can be plotted between this range of velocities and the distances between shot-point and the first geophone that record head waves (Figures 4 and 5).

The two Figures 4 and 5 represent the relationship between the distance between the shot-point and the first geophone which can record the head waves and the different velocity range of the second layer in case of the thickness of the first layer is fixed and equal to 1 meter and also 2 meters. It is clear that in the case of small differences between the two velocities of the dividing interface, the required distance between shot-point and the first geophone that should

Figure 4: The Relationship Between the Distance Between the Shot-Point and the First Geophone, Which Can Record the Head Waves, and the Different Velocity Range of the Second Layer in Case of the Thickness of the First Layer is Fixed and Equal to 1 Meter

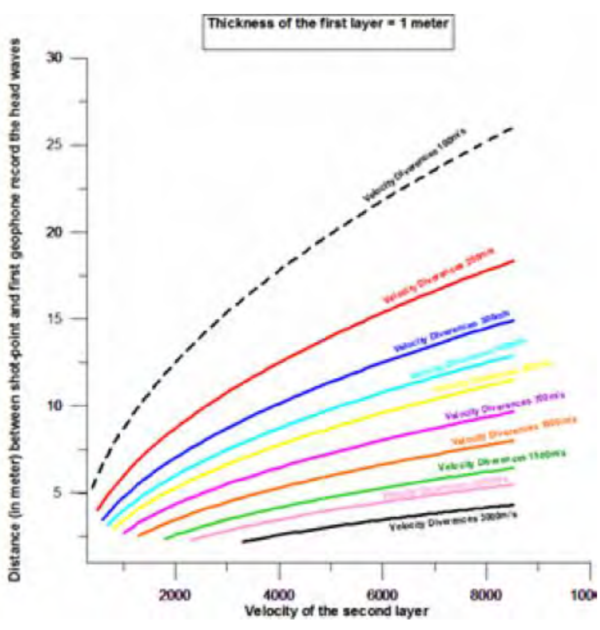


Figure 5: The Relationship Between the Distance Between the Shot-Point and the First Geophone, Which Can Record the Head Waves and the Different Velocity Range of the Second Layer in Case of the Thickness of the First Layer, is Fixed and Equal to 2 Meter

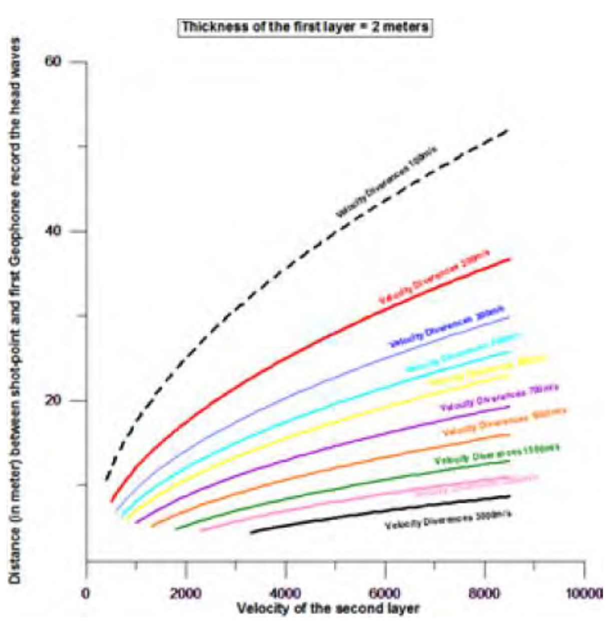


Figure 6: Main Relation Between (Velocities of First and Second Layers V_2/V_1), and (First Layer's Thickness and Shortest Distance Between Shot-Point and First Geophone Which Can Record Head Waves X_{dn}/h_1)

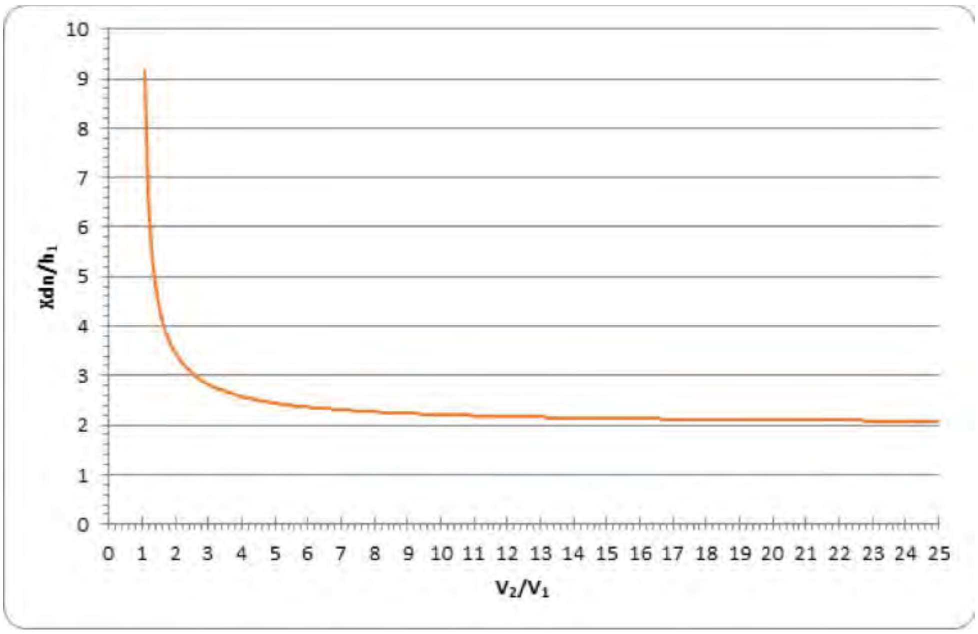
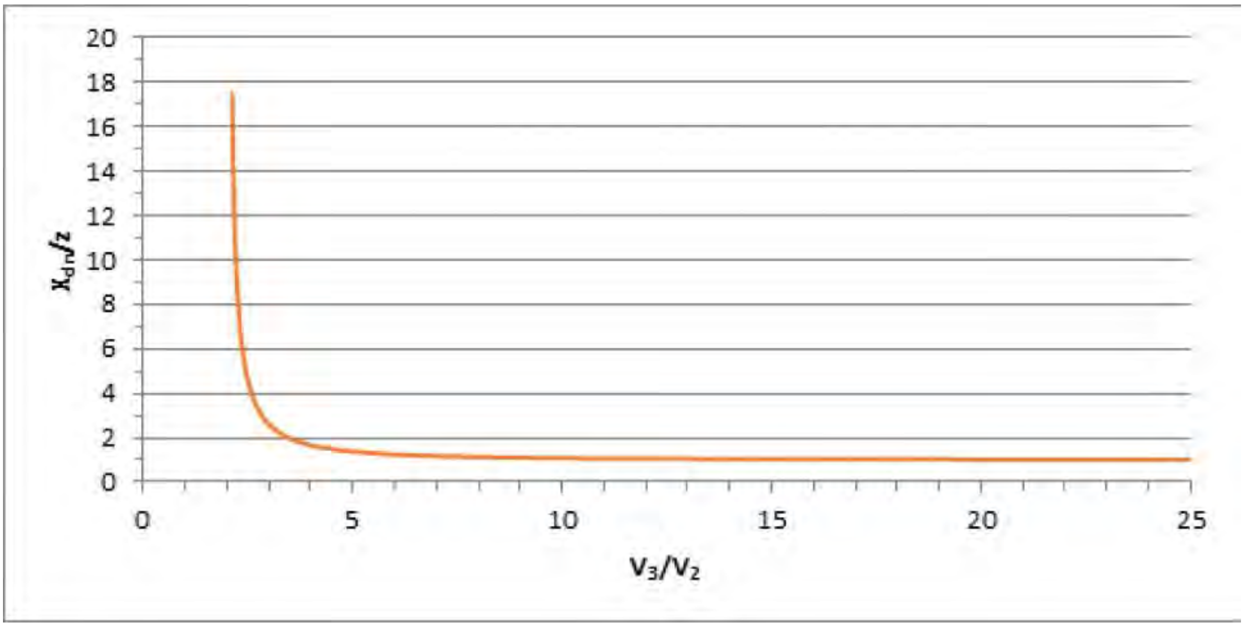


Figure 7: Main Relation Between (Velocities of Second and Third Layers V_3/V_2), and (Thickness of First and Second Layers and Shortest Distance Between Shot-Point and First Geophone Can Record Head Waves X_{dn}/Z)



record the arrival head waves is greater than in the case of large differences between the two velocities. This distance is constantly increasing when the difference is fixed by increasing the value of the two velocities. It is also deceptive that the higher the thickness of the first layer by one meter, the greater the distance by the distance equivalent to the meter in the first shape and in accordance with the velocity difference.

This relationship in Figure 6 shows that the largest distance between the source of the waves (shot-point) and the nearest geophone that can record the refracted head waves should not be more than nine times of the first layer's thickness. This occurs when the wave's velocity of the second layer is very close to the wave's velocity of the first layer. If the wave's velocity of the second layer is three times of the first layer's velocities, the required distance will be less than three times of the first layer's thickness. The relationship also indicates that the total penetrated depth not only depends on the length of the distance between the first and the last geophones but also it depends on the differences of velocities in the layers. In general the xd in this case can be determined by equation:

$$X_d = \{-0.0509\ (V_2/v_1) + 3.0221\}.h_1 \quad ...(E10)$$

Such relationship in Figure 7 shows that the largest distance between the source of the waves (shot-point) and the nearest geophone, which can sense the refracted head waves, should not be more than 18 times of the layers' thickness above the refraction surface. This occurs when the wave velocity's value in the third layer is very close to its velocity's value in the first layer. This distance is gradually reduced when the difference in velocities increases. If the wave velocity's value of the third layer is five times of its velocity in the second layer, the distance between the shot-point and the first geophone required will be less than twice of the refractive surface's depth. For three layer cases the xd can be determined by the next equation:

$$X_{dn} = \{-0.144(V_3/V_2) + 3.1622\}. Z \quad ...(E11)$$

CONCLUSION

When velocities above and below the reflection surface are equal, Irrespective of layers' number, the effective aspect of the distance between shot-point and first geophone, which can sense the head waves, is the depth from the surface, where the waves are created, to the reflection surface.

When there are minor dissimilarities between the two velocities of the dividing interface, the requisite distance between shot-point and the first geophone, which can detect the arrival head waves, is larger than in the case of major dissimilarities between the two velocities. This distance is continuously increasing if the variance is stable by increasing the value of the two velocities.

In the case of two layers, the maximum distance between the shot-point and the closest geophone, which can record the refracted head waves, should not be over nine times of the thickness of the first layer. This distance is doubled twice for each layer which is more than two subsurface layers upper the surface of refraction. This distance is gradually lessened with the difference of the increases of velocities.

Not only does the entire penetrated depth rely on the distance among the shot point, the first and the last geophones, but also it relies on the dissimilarities of velocities between the layers.

The results also show that the velocity differences affect the overall depth resulting from modeling. It is recommended to develop a mathematical perception of this effect and test it in a field.

REFERENCES

1. Alan Meulenbroek and Steve Hearn (2011), "Analysis of Converted Refractions for Shear Statics and Near-Surface Characterisation", *Exploration Geophysics,* Vol. 42, No. 2, pp. 147-154, DOI: 10.1071/EG10031.
2. Andrey V Masjukov and Igor S Murzov (2013), "Short-Period Static Correction Without Normal Move Out", *Geophysical Prospecting*, Vol. 61, No. 1, pp. 92-103, DOI: 10.1111/ j.1365-2478.2012.01050.x.
3. Burger H R (1996), "Exploration Geophysics of the Shallow Subsurface", p. 489, Prentice Hall.
4. Butlera S L and Sinhaa G (2014), Department of Geological Sciences, University of Saskatchewan, Canada.
5. Derecke Palmer (1981), "An Introduction to the Generalized Reciprocal Method of Seismic Refraction Interpretation", *Geophysics*, Vol. 46, No. 11, pp. 1508-1518, DOI:10.1190/1.1441157.
6. Dijksterhuis Fokko (2004), "Lenses and Waves: Christiaan Huygens and the Mathematical Science of Optics in the Seventeenth Century", Springer, ISBN: 1-4020-2697-8.
7. Fagin S W (1991), "Seismic Modeling of Geologic Structure: Tulsa, OK, Society of Exploration Geophysicists", *Geophysical Development Series*, Vol. 2.
8. Fowler C M R (1990), "The Solid Earth—An Introduction to Global Geophysics", p. 472, Cambridge University Press.

9. Gabriela Lara, Federico Lince Klinger, Laura Perucca, Guillermo Rojo, Nicolás Vargas and Flavia Leiva (2017), "Multiple Geophysical Methods Examining Neotectonic Blind Structures in the Maradona Valley, Central Precordillera (Argentina)", *Tectonophysics*, Vols. 712-713, pp. 634-642, DOI:10.1016/j.tecto.2017.06.021.

10. Hawkins L V (1961), "The Reciprocal Method of Routine Shallow Seismic Refraction Investigations", *Geophysics*, Vol. 26, No. 6, pp. 806-819, DOI: 10.1190/1.1438961.

11. Hilterman F J (1970), "Three-Dimensional Seismic Modeling: Geophysics", Vol. 35, pp. 1020-1037, DOI:10.1190/1.1440140.

12. Hobson George D, Hunter J A M and Scott J S (1970), "A Shallow Seismic Survey, Elastic Constants Studies and Surficial Geology of Part of Defence Research Establishment Suffield (DRES) Suffield, Alberta", Ottawa Department of Energy, Mines and Resources, Geological Survey of Canada, pp. 69-13.

13. Hubral P (1977), "Migration: Some Ray Theoretical Aspects: Geophysical Prospecting", Vol. 25, pp. 739-745.

14. Kearey P and Brooks M (1984), "An Introduction to Geophysical Exploration", p. 296, Blackwell Scientific Publications.

15. Kelley K R, Ward R W, Treitel S and Alford R M (1976), "Synthetic Seismograms—A Finite Difference Approach: Geophysics", Vol. 41, pp. 2-27, DOI:10.1190/1.1440605.

16. Shti Velman V (2003), "Application of Shallow Seismic Methods to Engineering, Environmental and Groundwater Investigations", *Bollettino di Geofisica teorica ed applicata,* Vol. 44, Nos. 3-4, pp. 209-222.

17. Steve Hearn and Alan Meulenbroek (2011), "Ray-Path Concepts for Converted-Wave Seismic Refraction", *Exploration Geophysics*, Vol. 42, No. 2, pp. 139-146, DOI: 10.1071/EG10030.

18. Susana P Vilanova, João Narciso, João P Carvalho, Isabel Lopes, Mário Quinta Ferreira, Carlos C Pinto, Rui Moura, José Borges and Eliza S Nemser (2018), "Developing a Geologically Based VS30 Site, Condition Model for Portugal: Methodology and Assessment of the Performance of Proxies", *Bulletin of the Seismological Society of America*.

19. Vrettos C (1996), "Soil Dynamics and Earthquake Engineering", *ELSEVIER*, Vol. 15, No. 6, pp. 381-386, DOI: 10.1016/0267-7261(96)00012-7.