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Correlation between Static and Dynamic Young's Modulus of Elasticity of Different Types of Rocks

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***Abstract*—Correlation between static and dynamic elastic modulus in rocks has often been addressed in various studies. In this study, the correlation between the static and dynamic elastic modulus of six different sedimentary, metamorphic and igneous rock types namely coarse-grained sandstone, fine-grained sandstone, schist, quartzite, basalt and granite are studied. Physical properties such as bulk density and porosity are evaluated of all samples. Static elastic modulus is determined by uniaxial compression test and dynamic elastic modulus is found out by ultrasonic pulse velocity testing. Scanning Electron Microscope experiment, X-Ray Diffraction experiment are also conducted on all the samples for mineralogical and petrological studies. The individual correlation between the static and dynamic elastic modulus of all the rock types have been proposed using linear regression with zero intercept and high values of coefficient of determination. These equations are useful in calculating the static elastic modulus using non-destructive techniques, in a wide range of rock materials. Finally, a general linear equation has been developed to relate ratio of static and dynamic elastic modulus with density of the rock.**

***Keywords*—correlation, static young's modulus, dynamic young's modulus, non-destructive testing, rock**

I. INTRODUCTION

Young's modulus is a basic geomechanical parameter that defines mechanical behaviour of rocks. This elastic modulus determines the material's deformability under active stress, which makes it an important parameter for any rocks [1–3]. Generally, deformability evaluation involves preparation of rock samples and application of load mechanically to them. However, the destructive nature of this test makes it not appropriate for use in certain circumstances, for example in historical buildings. The elastic modulus can also be computed from non-destructive testing by analysing outcome of experiments measuring the velocity of propagation of ultrasonic elastic waves, known as the dynamic modulus (E_d) [3–8]. The static modulus (E_s), obtained from traditional mechanical laboratory procedures, is necessary for calculating or modelling a building's deformations under in-service loading. In cases where the deformation characteristics of the rock cannot be calculated using destructive methods, the use of non-destructive techniques by mobile devices constitutes an acceptable alternative [5]. The dynamically determined elastic modulus is usually higher than the statically determined modulus, and the values vary significantly in rocks with a weak elastic modulus [6]. The disparity between the static and dynamic modulus is discussed in several studies, taking into account the effects of porosity, crystalline structure, and fracture or bedding plane spatial orientation on both testing methodologies [7–9]. Although, elastic wave speeds are extremely sensitive to micro-cracks, but the static method used to measure the deformability of the rock is slightly more sensitive to discontinuity [9, 10]. Table I summarizes the relationships between the static and dynamic elastic modulus for various rock types and value ranges suggested by several authors [3–16].

TABLE I. SUMMARY OF RELATIONSHIPS

Relationship	**R^2**	**E_d (GPa)**	**Rock type**	**Reference**
$E_s = 1.26E_d - 29.5$	0.82	40–120	Igneous-metamorphic	[7]
$E_s = a\,E_d^{\,b}$ a [0.097–0.152] b [1.485–1.388]	–	20–135	Sandstone-granite	[15]
$E_s = 0.74E_d - 0.82$	0.70	5–130	All types	[3]
$\mathrm{Log}_{10}E_s = 0.77 \log_{10}(\rho\mathrm{bulk}E_d) + 0.02$	0.92	5–130	All types	[3]
$E_s = 1.05E_d - 3.16$	0.99	25–110	All types	[5]
$E_s = 0.018E_d^2 + 0.422\,E_d$	–	–	Sedimentary	[14]
$E_s = 1.153E_d - 15.2$	–	–	Es > 15GPa	[12]
$E_s = 0.076V_p^{3.23}$	–	–	Shale	[13]
$E_s = E_d/3.8\alpha s - 0.68$	–	5–50	Limestone-marble	[11]
$E_s = 0.867E_d - 2.085$	0.96	5–30	Calcarenite	[16]
$\mathrm{Log}_{10}E_s = 1.28\log_{10}(\rho_{\mathrm{bulk}}E_d) - 4.71$	0.97	5–30	Calcerenite	[16]
$E_s = 0.014E_d^{1.96}$	0.87	13–74	Limestone	[8]
$E_s = 0.169V_p^{3.324}$	0.90	13–74	Limestone	[8]

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The main objective of this study is to develop a correlation in order to obtain the static elastic modulus of variety of rocks of different origins (a wide range of hard and soft rocks) by non-destructive ultrasonic testing, with broader range of elastic modulus (i.e., 10 to 100 GPa). Its noteworthy from previous research as shown in Table I that when E_d is zero, the E_s is a non-zero number which is infeasible. Therefore, in this study the intercept of linear equation is considered zero.

II. MATERIAL AND THEIR CHARACTERISTICS

In this study, six different types of rock of different origins (i.e., igneous, sedimentary and metamorphic) are selected. The rocks are namely Coarse-Grained Sandstone (CGS), Fine-Grained SANDSTONE (FGS), schist, basalt, granite and quartzite. The appearance of the rocks tested in this investigation is shown in Fig. 1 at mesoscopic scale.

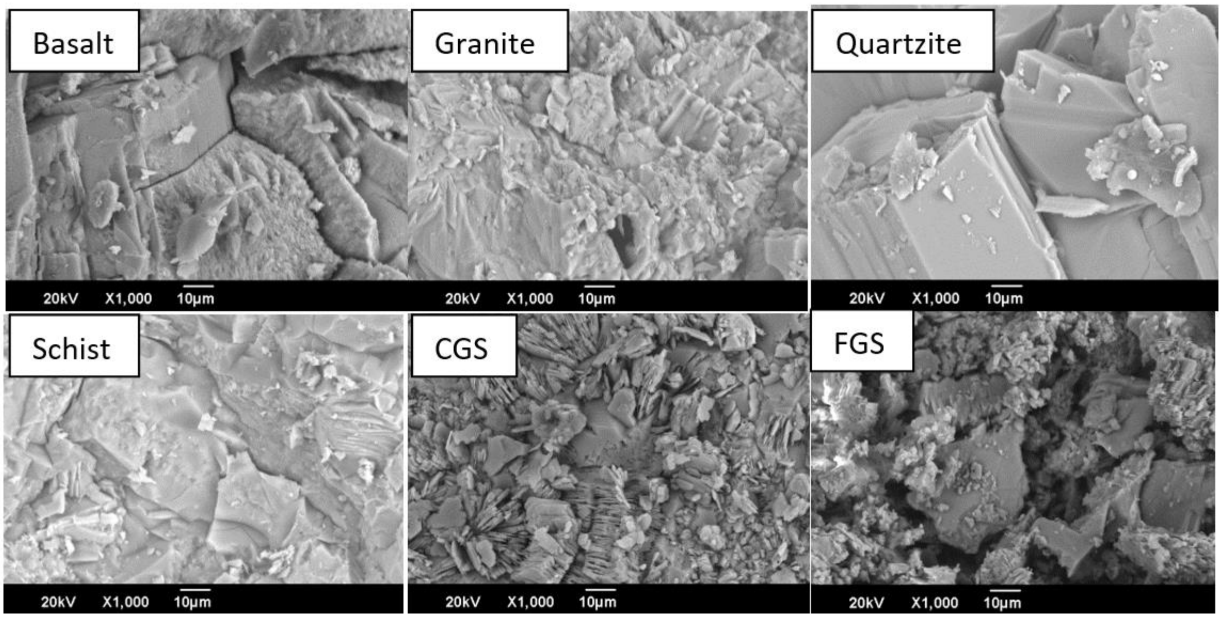

Fig. 1. SEM micrographs of rock sample.

The petrography of these rock samples in Fig. 1 shows that all rock samples have a homogenous texture that is free from fracture or cavities.

A brief petrologic description is also worked out based on the X-Ray diffraction experiment.

It is observed that basalt sample consists of andesine as its major constituent with little traces of clinopyroxene and hedenbergite.

The granite sample is composed of with muscovite, quartz and clinoclore as major mineral. The main component in the quartzite rock sample is ferroan with small quantities of zeolite and old hamite. The predominant minerals in the schist sample are kaolinite, quartz and goethite. The sandstone sample is mainly characterized by kaolinite minerals, in addition to small quantities of orthoclase, quartz and zeolite.

III. EXPERIMENTAL METHODOLOGY

In this investigation various test are performed on the prepared rock core samples to ascertain bulk density, total porosity, P and S ultrasonic velocity and uniaxial compressive strength. For experimentation, 30 cylindrical samples (five of each rock type) 47 mm in diameter and 94–96 mm long are prepared as mentioned in Table II. Cylindrical samples of basalt are 63 mm in diameter and 122–124 mm in length. The aspect ratio of at least 2 is maintained as per relevant testing standards.

A. Porosity

Porosity is a physical property of a rock which is the representation of void spaces in a rock. It is defined as the ratio of the volume of voids in rock to the total volume of rock. Porosity can be represented in percentage form or decimal form from 0 to 1. The interconnection of these pore spaces creates a pore system which results in decreased strength parameters. Bulk density also gives a representation of porosity. Higher the bulk density, higher the compaction, lower the pore spaces and vice versa. The average values of each rock type are shown in Table II [17].

B. Ultrasonic Testing

The Ultrasonic Pulse Velocity (UPV) test is a measure of the quality of rock. It is a non-destructive test which measures the rock strength and quality of rock. It makes an ultrasonic wave to pass through rock of known length and measures the velocity of the ultrasonic wave sent. The time taken by the pulse to pass through the considered length of rock is measured. A higher value of velocity indicates a better quality of rock and lower value indicates voids or cracks in the rock. The testing equipment consists of an ultrasonic pulse generation circuit which is generally electric and a transducer which transforms the generated electric wave to a mechanical wave with an oscillation frequency of range between 40 kHz and 50 kHz. The pulse is received using pulse reception circuit. The most common type of UPV testing instrument is the PUNDIT LAB (Portable Ultrasonic Non-destructive Digital Indicating Tester). The PUNDIT has the capacity to measure the speed of propagation of an ultrasonic longitudinal stress wave. The uniformity of the rock can be found out using this test. The location of internal voids, extent of void and severity of deterioration can be evaluated.

C. Uniaxial Compressive Strength (UCS)

One of the most important strength properties of the rock or soil sample is its resistance against compression which is determined by performing the UCS. Rocks are elasto-plastic in nature which induces brittleness to the rock when once the load coming over it is more than its capacity. This particular strength at which the specimen breaks due to application of load is called the UCS of the rock. It is mathematically represented as the load applied at failure by the cross-sectional area of the specimen.

IV. RESULTS AND DISCUSSION

Table II shows the physical and mechanical characteristics of six rock types under the study. It may be observed that the sedimentary rock (coarse and fine-grained sandstone) exhibited higher porosity, which can be substantiated by its relatively lower apparent density. Basalt has more porosity than the sedimentary rocks. Then metamorphic rocks (schist and quartzite) show less porosity. The hard rock appeared to have lowest porosity. Schist and coarse-grained sandstone showed the highest and lowest compressive strength respectively. Furthermore, the dynamic modulus was found higher than the static modulus in all cases. The maximum and minimum modulus of elasticity are observed in quartzite and coarse-grained sandstone respectively.

TABLE II. SUMMARY OF PHYSICAL AND MECHANICAL PROPERTIES OF THE ROCK SAMPLES

Type of rock	**Bulk density (gm/cc)**	**Porosity**	**Uniaxial compressive strength (MPa)**	**Static Young's modulus (E_s) (GPa)**	**Dynamic Young's modulus (E_d) (GPa)**
Coarse grained sandstone	1.92	23.33	8.20	8.60	10.51
Fine grained sandstone	2.11	15.05	15.98	9.82	21.80
Basalt	2.75	14.33	33.43	20.77	61.25
Quartzite	2.66	1.11	51.89	56.70	100.49
Schist	2.73	3.01	72.43	45.31	87.76
Granite	2.83	0.28	55.17	50.45	97.03

Fig. 2 shows the general increasing trend of static modulus of rocks with increase in dynamic modulus of the same.

Moreover, Figs. 3–8 give the individual relationship between static and dynamic elastic modulus of rocks under investigation. It can be seen that all linear equations are obtained with high coefficient of determination. Table III gives the relationship between static and dynamic elastic modulus of various rocks considered under this study. It can be observed that all linear equations are obtained with high coefficient of determination.

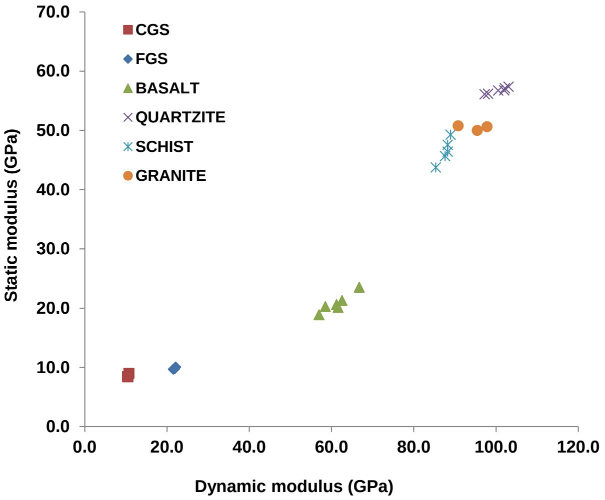

Fig. 2. Static vs. dynamic Young's modulus of all rock samples.

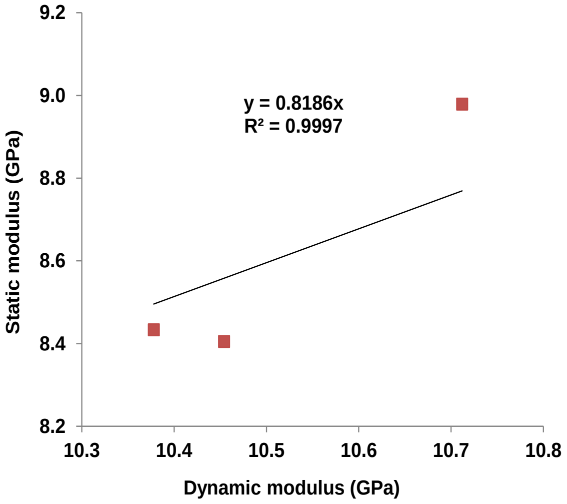

Fig. 3. Static vs. dynamic Young's modulus for coarse grained sandstone.

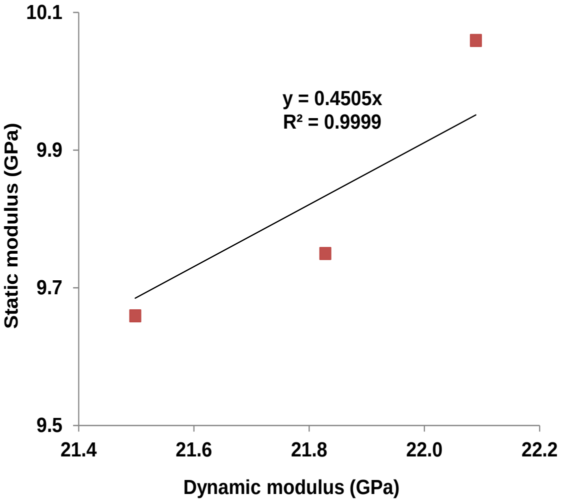

Fig. 4. Static vs. dynamic Young's modulus for fine grained sandstone.

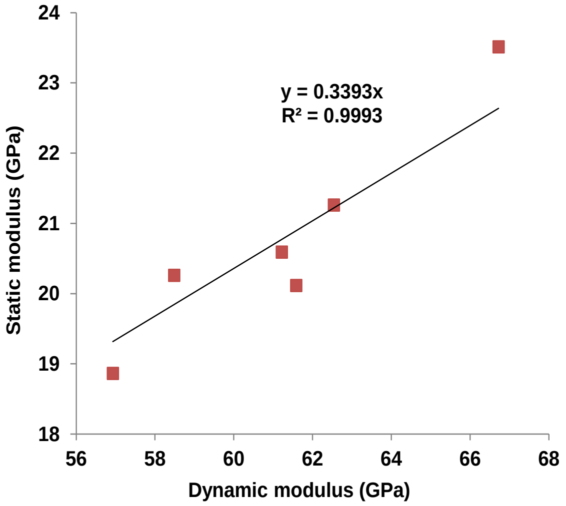

Fig. 5. Static vs. dynamic Young's modulus for basalt.

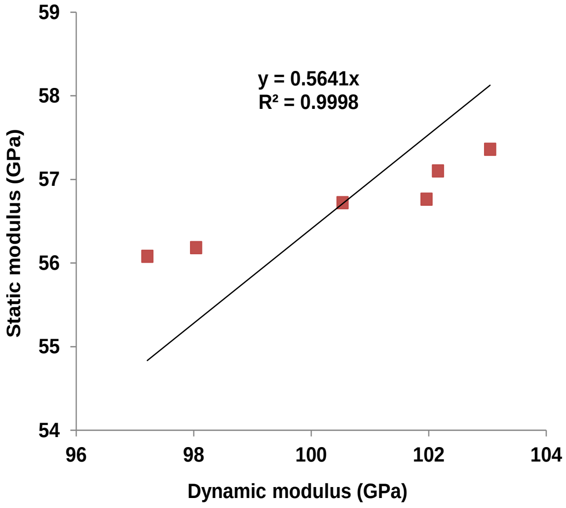

Fig. 6. Static vs. dynamic Young's modulus for quartzite.

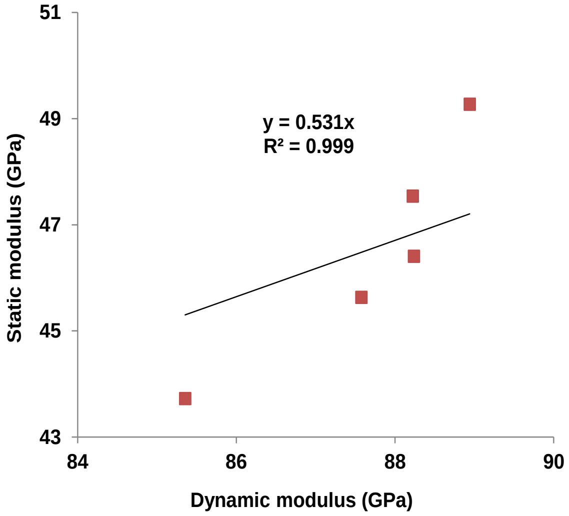

Fig. 7. Static vs. dynamic Young's modulus for schist.

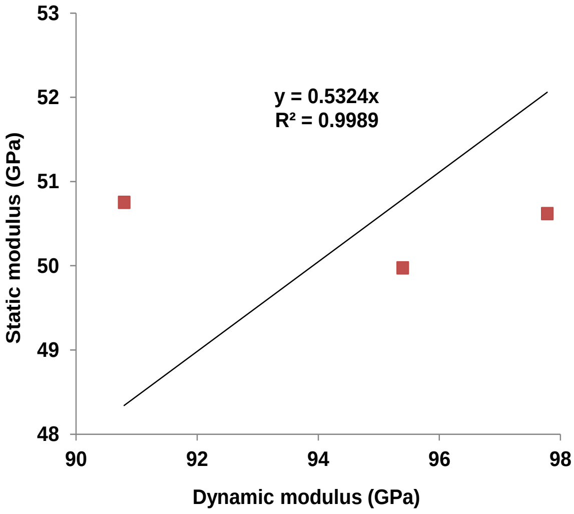

Fig. 8. Static vs. dynamic Young's modulus for granite.

TABLE III. SUMMARY OF CORRELATIONS BETWEEN STATIC AND DYNAMIC ELASTIC MODULUS

Rock sample	**Correlation between static and dynamic Young's Modulus**	**R^2**
Coarse grained sandstone	$E_s = 0.8186E_d$	0.99
Fine grained sandstone	$E_s = 0.4505E_d$	0.99
Basalt	$E_s = 0.3393E_d$	0.99
Quartzite	$E_s = 0.5641E_d$	0.99
Schist	$E_s = 0.531E_d$	0.99
Granite	$E_s = 0.5324E_d$	0.99

Table IV gives the relationship between static and dynamic elastic modulus of various rocks considered under this study. It can be seen that all linear equations are obtained with high coefficient of determination.

TABLE IV. SUMMARY OF CORRELATIONS BETWEEN STATIC AND DYNAMIC ELASTIC MODULUS

Rock sample	**Slope of equations (E_s/E_d)**	**Bulk density (gm/cc)**
Coarse grained sandstone	0.82	1.92
Fine grained sandstone	0.45	2.11
Basalt	0.33	2.66
Quartzite	0.56	2.73
Schist	0.53	2.75
Granite	0.53	2.83

Fig. 9 exhibits the relation derived between ratio of static and dynamic elastic modulus with density of the rock.

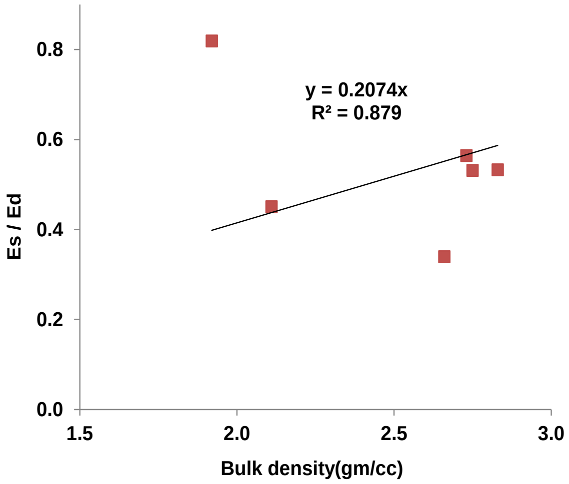

Fig. 9. Es/Ed vs Bulk density of rock.

V. Conclusion

In this study, a wide variety of intact rocks material were analyzed and correlations were obtained. The static elastic modulus tests are basically destructive type of test. The obtained correlations can be reasonably used to find the static modulus without destructing the sample. The rock samples whose elastic modulus is lower had higher porosity and less bulk density and the rock sample whose elastic modulus is higher had lower porosity and higher density. The differences in the static and dynamic values are ascribed to effect of porosity, size and spatial orientation of cracks or bedding planes on both unique measuring techniques. In static testing, higher porous samples take initial load for rearrangement and filling cavities thereby resulted in higher difference in dynamic and static elastic modulus. This study proposes different correlations established from rocks having dynamic modulus ranged from 10 to 100 GPa, including rocks of igneous, sedimentary and metamorphic origin. All the rocks exhibited an exceptionally homogeneous texture (free from cracks, voids, etc.), which indicates that the petrophysical variations among the rocks were ascribed to: (i) mineralogical variations, (ii) grain size differences, (iii) porosity differences, and (iv) pore size differences. Porosity has mostly affected the mechanical behavior of the rocks specially in sedimentary rock type i.e., coarse grained sandstone and fine-grained sandstone.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

SSR has done the experimental works and wrote the paper with the help of SP after analyzing results. RDL contributed the work by reviewing the paper. All authors had approved the final version.

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